Math 215 – Problem Set 1

Three Dimensional Coordinate Systems, Vectors, Dot Product, Cross Product, Lines and Planes

Math 215 SI
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1 Review

1.1 Three-Dimensional Coordinate Systems

Points in three-dimensional space are written as ordered triples (x, y, z). The x-, y-, and z-axes are mutually perpendicular.

1.2 Vectors

A vector in $\mathbb{R}^3$ can be written as $\langle a, b, c\rangle$. Vectors have magnitude and direction and can be added or multiplied by scalars.

1.3 Dot Product

For vectors $\mathbf{a} = \langle a_1, a_2, a_3\rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3\rangle$,

$$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3.$$

The dot product is used to find angles and test orthogonality:

$$\cos(\theta) = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}.$$

1.4 Cross Product

The cross product $\mathbf{a} \times \mathbf{b}$ is a vector perpendicular to both $\mathbf{a}$ and $\mathbf{b}$:

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}.$$

1.5 Lines and Planes

A line is written parametrically using a point and a direction vector. We typically write it in the form:

$$r(t) =< at + a_0, bt + b_0, ct + c_0 >$$

A plane is written in its vector form using a point and a normal vector that is orthogonal to every vector in the plane. We typically write it in the form:

$$\mathbf{n} \cdot (\mathbf{r} - \mathbf{r_0}) = 0$$

Which can be simplified into the linear equation expression of a plane given by:

$$ax + by + cz + D = 0$$

2 Problems

Problem 1

Suppose P_1 is a plane that contains the points $(0, 3, 0),\ (-3, 0, 0),$ and $(2, 6, 1)$

(a) Find the equation of the plane P_1.

(b) Let P_2 be the plane that contains the point $(0, 2, 1)$ and the line

$$\ell(t) = \langle 2t,\ t,\ 1 + 3t \rangle.$$

Find the angle between P_1 and P_2.

Problem 2

Let $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$. Let $\theta_1, \theta_2, \theta_3$ be the angles that $\mathbf{v}$ makes with the x-, y-, and z-axes. Find

$$\cos^2(\theta_1) + \cos^2(\theta_2) + \cos^2(\theta_3).$$

Problem 3

A cube has side length 6. Segments $\overline{ab}$ and $\overline{pq}$ intersect at the center c of the cube. Let T be the triangle with vertices a, c, and q.

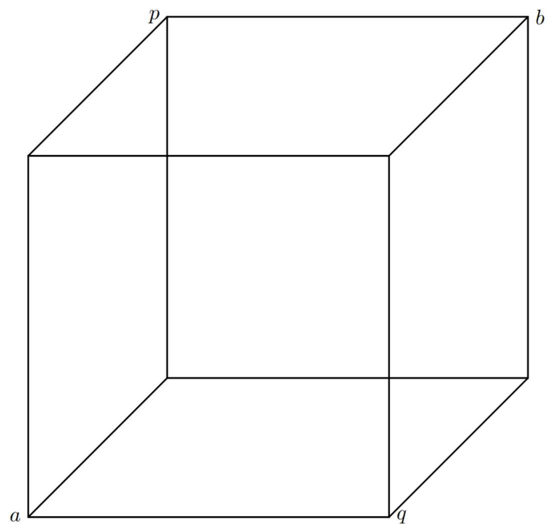

Figure 1: Cube with points a, b, p, and q.

(a) Find the area of triangle T.
(b) If θ is the angle of T at c, find $\cos(\theta)$.