Math 215 – Problem Set 3

Partial Derivatives, Tangent Planes, Linear Approximations

Math 215 SI
February 1, 2026

3 Review

3.1 Functions of Several Variables

A function of two variables is a rule that assigns to each ordered pair (x, y) in a domain D a unique real number $f(x, y)$.

- **Level Curves:** A level curve of a function $f(x, y)$ is the set of all points (x, y) in the domain of f such that $f(x, y) = k$, where k is a constant (in the range of f).
- **Contour Map:** A collection of level curves for various values of k is called a contour map. These curves show where the "height" of the surface is constant.
- **Level Surfaces:** For a function of three variables $w = f(x, y, z)$, the level surface is given by $f(x, y, z) = k$.

3.2 Partial Derivatives

The partial derivative of $f(x, y)$ at a point (a, b) is defined as the rate of change of the function as only one variable varies:

$$f_x(a,b) = \frac{\partial f}{\partial x}(a,b) = \lim_{h\to 0} \frac{f(a+h,b) - f(a,b)}{h}$$

$$f_y(a,b) = \frac{\partial f}{\partial y}(a,b) = \lim_{h\to 0} \frac{f(a,b+h) - f(a,b)}{h}$$

Practical Advice: Calculate partial derivatives by treating the other variable(s) as constants. **Theorem:** If f_{xy} and f_{yx} are both continuous, then $f_{xy} = f_{yx}$. This is usually the case in this class.

3.3 Tangent Planes and Linear Approximations

For a differentiable function $f(x, y)$, the tangent plane to the graph $z = f(x, y)$ at the point (a, b) is given by:

$$z = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

A linear approximation is using the tangent plane to approximate values of $f(a, b)$ such that:

$$f(x,y) \approx f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

2

Problems

Problem 1

Find or estimate, depending on the data provided, the partial derivative in the x direction at the point $(0, 0)$ and the y direction at the point $(0, 0)$ for each of the following functions.

(i) For a function given by the formula $f(x, y) = y^2 \cos(1 + x - y^2 x)$

(ii) For a function g described by the data in the table below.

$x \setminus y$	-2	-1	0	1	2
-2	6	9	9	9	10
-1	12	16	18	19	20
0	20	22	25	27	30
1	28	36	43	47	48
2	35	49	55	61	66

Problem 2

Suppose $g(x, y) = x + \ln(5x^2 - 4y^2)$. Find an equation for the tangent plane to the surface given by the equation $z = g(x, y)$ at the point $(1, 1, 1)$.

Problem 3

Suppose that $f(x, y)$ is a differentiable function with continuous derivatives with:

$$f(2,5) = 7, \quad f_x(2,5) = 3, \quad f_y(2,5) = -2$$

Consider the curve C given by the intersection of the plane $x = 2$ and the surface $z = f(x, y)$. Find a parametric equation of the line that lies on the plane $x = 2$ and is a tangent line to C at point $(2, 5, 7)$.

Problem 4

Which of the following equations does the function $z = f(x+t)+g(x-t)$ satisfy for all differentiable functions $f(s)$ and $g(s)$ in a single variable?

(a) $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial t} = 0$

(b) $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial t}$

(c) $\frac{\partial z}{\partial x} + \frac{\partial^2 z}{\partial t^2} = 0$

(d) $\frac{\partial z}{\partial x} = \frac{\partial^2 z}{\partial t^2}$

(e) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial t^2} = 0$

(f) $\frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 z}{\partial t^2}$