Math 215 – Problem Set 5

Applications of Double Integrals, Triple Integrals, and Cylindrical Coordinates

Math 215 SI
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4 Review

4.1 Applications of Double Integrals

Mass Calculation: Consider a lamina occupying a region D in the xy-plane with density $\rho(x, y)$. The mass is given by

$$M = \iint_D \rho(x, y)\, dA$$

Center of Mass: If the density is $\rho(x, y)$ and total mass is M, then

$$\bar{x} = \frac{1}{M} \iint_D x\, \rho(x, y)\, dA$$
$$\bar{y} = \frac{1}{M} \iint_D y\, \rho(x, y)\, dA$$

Symmetry can often simplify these calculations.

4.2 Triple Integrals

Let $f(x, y, z)$ be defined on a solid region B. The triple integral of f over B is

$$\iiint_B f(x, y, z)\, dV$$

Using Fubini's Theorem, this can be written as an iterated integral such as

$$\int_a^b \int_c^d \int_e^f f(x, y, z)\, dx\, dy\, dz$$

Useful Applications:

- Volume of a solid:
$$\iiint_B 1\, dV$$
- Mass with density $\rho(x, y, z)$:
$$M = \iiint_B \rho(x, y, z)\, dV$$
- Center of mass:
$$\bar{x} = \frac{1}{M} \iiint_B x\, \rho(x, y, z)\, dV$$
$$\bar{y} = \frac{1}{M} \iiint_B y\, \rho(x, y, z)\, dV$$
$$\bar{z} = \frac{1}{M} \iiint_B z\, \rho(x, y, z)\, dV$$

Always check for symmetry to determine if integrals evaluate to zero.

4.3 Triple Integrals in Cylindrical Coordinates

Coordinate relationships:

$$x = r\cos\theta, \quad y = r\sin\theta, \quad z = z$$

Jacobian:

$$dV = r\,dr\,d\theta\,dz$$

If $f(x, y, z)$ is continuous on a solid E, then

$$\iiint_E f(x, y, z)\,dV = \iiint_E f(r\cos\theta, r\sin\theta, z)\;r\,dr\,d\theta\,dz$$

Cylindrical coordinates are especially useful for regions with circular symmetry.

Problems

Problem 1

Evaluate the triple integral for the following choices of B:

$$\iiint_B xyz \, dV$$

(i) B is the cube $0 \leq x \leq 1,\ 0 \leq y \leq 1,\ 0 \leq z \leq 1$
(ii) B is the region $0 \leq x \leq y \leq z \leq 1$

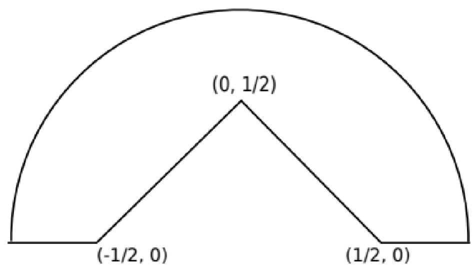
Problem 2

(i) Consider the half-disc defined by

$$0 \leq x^2 + y^2 \leq 1, \quad y \geq 0$$

Assume the density is $\rho(x, y) = 1$. Find $\bar{y}$, the y-coordinate of the center of mass of the half-disc.

(ii) Find the y-coordinate of the center of mass of the triangular region with the vertices shown in the figure, assuming the same density as in part (i).



Problem 3

Find the volume of the solid bounded by the following two paraboloids using cylindrical coordinates:

$$z = x^2 + y^2$$

$$z = 2 - x^2 - y^2$$