MATH 215 SI

Session 2

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Lectures 5 - 8

- 12.6 Cylinders and Quadric Surfaces
- 13.1 Vector Functions and Space Curves
- 13.2 Derivatives and Integrals of Vector Functions
- 13.3 Arc Length and Curvature
- 13.4 Motion in Space: Velocity and Acceleration

12.6 Cylinders and Quadric Surfaces

A cylinder is a surface that consists of all lines (rulings) parallel to a given line and passing through a given plane curve.

- If one variable is missing from an equation of a surface, then the surface is a cylinder with rulings parallel to that variable's axis
- A quadric surface is the graph of a second-degree equation in x, y, z
- To identify a quadric surface, find its traces (intersections with planes parallel to the coordinate planes)

12.6 Cylinders and Quadric Surfaces

cont.

The Standard Quadric Surfaces

- Ellipsoid: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$
- Elliptic paraboloid: $\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$
- Hyperbolic paraboloid $\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$
- Cone: $\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$
- Hyperboloid of one sheet: $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$
- Hyperboloid of two sheets: $-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Exercises

12.6 Cylinders and Quadric Surfaces

- What surface in $\mathbb{R}^3$ is represented by the equation $z = x^2$?
- Identify the surface given by $y = x^2 - z^2$ by finding its horizontal and vertical traces (with respect to y)

13.1 Vector Functions and Space Curves

A vector function $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$ assigns a vector to each value of the parameter t

- A vector function is a function whose domain is a set of real numbers and whose range is a set of vectors.
- $\lim_{t \to a} \mathbf{r}(t) = \left\langle \lim_{t \to a} f(t), \lim_{t \to a} g(t), \lim_{t \to a} h(t) \right\rangle$, provided the limits of the component functions exist
- $\mathbf{r}$ is continuous at a if $\lim_{t \to a} \mathbf{r}(t) = \mathbf{r}(a)$
- A space curve C is the set of all points $(f(t), g(t), h(t))$ for $t \in I$ for some interval I.

13.2 Derivatives and Integrals of Vector Functions

The derivative of a vector function is given componentwise
$\mathbf{r}'(t) = \lim_{h\to 0} \frac{\mathbf{r}(t+h)-\mathbf{r}(t)}{h} = \langle f'(t), g'(t), h'(t)\rangle$

- $\mathbf{r}'(t)$ is the tangent vector to the curve at the point defined by $\mathbf{r}(t)$, provided $\mathbf{r}'(t)$ exists and $\mathbf{r}'(t) \neq \mathbf{0}$
- Integration is also componentwise: $\int \mathbf{r}(t)\,dt = \langle \int f(t)\,dt, \int g(t)\,dt, \int h(t)\,dt\rangle$

13.2 Derivatives and Integrals of Vector Functions

cont.

Differentiation Rules

Let $\mathbf{u}$, $\mathbf{v}$ be differentiable vector functions, c a scalar, and f a scalar function.

- $\frac{d}{dt}[\mathbf{u}(t) + \mathbf{v}(t)] = \mathbf{u}'(t) + \mathbf{v}'(t)$
- $\frac{d}{dt}[c\mathbf{u}(t)] = c\mathbf{u}'(t)$
- $\frac{d}{dt}[f(t)\mathbf{u}(t)] = f'(t)\mathbf{u}(t) + f(t)\mathbf{u}'(t)$
- $\frac{d}{dt}[\mathbf{u}(t) \cdot \mathbf{v}(t)] = \mathbf{u}'(t) \cdot \mathbf{v}(t) + \mathbf{u}(t) \cdot \mathbf{v}'(t)$
- $\frac{d}{dt}[\mathbf{u}(t) \times \mathbf{v}(t)] = \mathbf{u}'(t) \times \mathbf{v}(t) + \mathbf{u}(t) \times \mathbf{v}'(t)$

Exercises

13.1 Vector Functions and Space Curves

13.2 Derivatives and Integrals of Vector Functions

The trajectory of a particle is given by $\mathbf{r}(t) = \langle \sqrt{3}t^2, 2t^3, \sqrt{6}t^2 \rangle$. Let C denote the corresponding space curve.

- Find an equation for the tangent line to C at the point $(4\sqrt{3}, 16, 4\sqrt{6})$

13.3 Arc Length and Curvature

- The length of a space curve $\mathbf{r}(t)$, $t \in [a,b]$ is given by:

$$L = \int_a^b |\mathbf{r}'(t)|\,dt = \int_a^b \sqrt{[f'(t)]^2 + [g'(t)]^2 + [h'(t)]^2}\,dt$$

- The curvature of a curve is defined by $\kappa(t) = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|}$ or (more usefully):

$$\kappa(t) = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$$

Exercises

13.3 Arc Length and Curvature

- Recall the last exercise. Now find the length of C on the interval $t \in [0, \sqrt{8}]$.

13.4 Motion in Space: Velocity and Acceleration

If a particle's position at time t is $\mathbf{r}(t)$, then

- Velocity: $\mathbf{v}(t) = \mathbf{r}'(t)$
- Speed: $|\mathbf{v}(t)| = |\mathbf{r}'(t)|$
- Acceleration: $\mathbf{a}(t) = \mathbf{v}'(t) = \mathbf{r}''(t)$

Exercises

13.4 Motion in Space: Velocity and Acceleration

In this problem all coordinates are measured in meters and time is measured in seconds. At time $t = 0$ a ladybug, named Sam, is at position $(1, 1, 1)$ and is flying with constant velocity $\langle 1, 2, 3 \rangle$ meters per second. A sensor placed at $(3, 6, 7)$ can detect ladybug motion that occurs within a sphere of radius 7 meters. Does the sensor detect Sam? If so, at what time is Sam last detected by the sensor?

Fin

This concludes session 2.